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REFLECTION OF SOUND AT AN INTERFACE, TAKING INTO ACCOUNT MASS AND HEAT TRANSPORT ACROSS IT

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Abstract—With the aid of an equation which was derived by Meinhold-Heerlein in 1973, five boundary conditions are given for a two-phase system. These boundary conditions include the two of classical acoustics, namely, steadiness of pressure and of the normal component of particle velocity across the interface. They permit a calculation of coefficients of reflection and transmission which obtain the classical acoustic coefficients as leading terms. Moreover, three transformation coefficients are given, describing the movement of the interface and the propagation of heat waves in the liquid and vapour. The time-averaged energy dissipation is calculated.

NOMENCLATURE

A dimensionless amplitude of the pressure wave
A_i, A_r, A_t dimensionless amplitude of the incident, reflected, and transmitted sound wave
a thermal diffusivity, $\lambda/\rho_0 c_p$
a_ρ ratio of densities, ρ_{g0}/ρ_{l0}
a_c ratio of sound velocities, c_g/c_l
B dimensionless amplitude of the particle velocity
C dimensionless amplitude of the heat wave
C_g, C_l dimensionless amplitude of the heat wave in the vapour and the liquid
c sound velocity
c_p specific heat at constant pressure
D_r, D_t acoustic coefficients of reflection and transmission
h specific enthalpy
$\mathbf{J}^\psi, J_E, J_M, \mathbf{J}^q$ fluxes of ψ, energy, mass, and heat
k wave number of sound wave
L_0 heat of condensation
$L_{EE}, L_{EM}, L_{ME}, L_{MM}$ Onsager coefficients
L_{12} $L_{ME}-h_{g0}L_{MM}$
L_{22} $L_{EE}-h_{g0}(L_{EM}+L_{ME})+h_{g0}^2 L_{MM}$
$\dot{m}$ mass flux
$\dot{m}_1$ mass flux in first approximation
P symbol for the interface
p pressure
Pr modified Prandtl number
s specific entropy
T temperature
t time
u specific internal energy
v particle velocity
W_g, W_l transformation coefficients of the heat wave in the vapour and the liquid
$\hat{w}$ dimensionless amplitude of the movement of P
Z transformation coefficient of the movement of P.

Greek symbols
α_p volume expansion coefficient
Γ $[p_0/(\rho_0 c^2)][(\gamma-1)/(\alpha_p T_0)]$
γ ratio of specific heats
$\boldsymbol{\delta}$ unit tensor
ε parameter of expansion
ζ bulk viscosity
η shear viscosity
κ wave number of heat wave
λ heat conductivity
μ chemical potential
$\mathbf{\Pi}$ Navier–Stokes stress tensor
$d\pi/dT$ vapour–pressure curve
ρ mass density
σ^ψ source density in the bulk
τ_ψ surface production of ψ
ψ general field function
χ_ψ surface density of ψ
ω circular frequency.

Subscripts and superscripts
g vapour
i imaginary part (as second index)
l liquid
r real part (as second index)
0 state of equilibrium
~ deviation from equilibrium
– time average.

1. INTRODUCTION

THE SCOPE of this paper is to give an estimate of the dissipation of energy occurring during the incidence of a sound wave on an interface P. It is assumed that there is mass and energy transport across P caused by condensation and evaporation. To that end there will be needed some coefficients which describe reflection and transmission of sound waves at P and their transformation into heat waves. Therefore, in Section 2, equations will be provided which describe the propagation of sound and heat waves in two-phase systems. In Section 3, five surface conditions will be derived with the aid of a general balance equation for interfaces, among them the two classical* ones, namely,

* Classical in the sense of the treatise on the problem by Rayleigh [1].

continuity of pressure and of the normal component of the velocity across P.

By means of the above-mentioned five surface conditions, acoustic coefficients will be evaluated in Section 4. Section 5, finally, contains the calculation of the time-averaged energy dissipation for a two-phase system consisting of pure water and its vapour.

2. DISPERSION OF SOUND AND TEMPERATURE WAVES

In this section we will consider a thermohydrodynamic system Σ which is not subjected to external forces and consists of two phases and one component, e.g. water and its vapour. Σ is divided into two partial systems Σ_g and Σ_l by an interface P which is infinitely extended in the x–y plane. Across P there is supposed to take place mass and energy transport; Σ_g and Σ_l have to conduct sound and heat waves, furthermore, both systems have to be built up in such a way that there is at least one finite volume element in the interior of each partial system which is not intersected by P. Thus, balance equations for mass, linear momentum, energy, and entropy hold in the interior of either of the two systems. These equations are either written in the form

$$\rho\frac{d\psi}{dt}+\mathrm{div}\,\mathbf{J}^{\psi}=\sigma^{\psi}, \tag{1}$$

or

$$\frac{\partial}{\partial t}(\rho\psi)+\mathrm{div}\,(\rho\psi\mathbf{v}+\mathbf{J}^{\psi})=\sigma^{\psi}. \tag{2}$$

Here ρ denotes the mass density, ψ an arbitrary field quantity $\psi(\mathbf{r},t)$ which is split into a part which is constant with respect to the local vector $\mathbf{r}$ and time t and a small deviation from it which depends on place and time

$$\psi(\mathbf{r},t)=\psi_0+\tilde{\psi}(\mathbf{r},t), \tag{3}$$

$\mathbf{J}^{\psi}$ denotes the flux density of ψ and σ^{ψ} the source density of ψ.

Let there be a sound wave, represented by its amplitude A_i, which is impinging on P out of the gas (cf. Fig. 1). The sound wave will be partially reflected (with A_r) and partially transmitted (with A_t); furthermore, there will take place condensation and evaporation in time with the sound wave, which makes (a) P oscillating around its position of rest at $z=0$ and (b) Σ_g and Σ_l generating heat waves with the amplitudes C_g and C_l caused by setting free heat of evaporation. Equations describing the above-mentioned problem are:

(1) the balance of mass (M)

$$\frac{\partial}{\partial t}\rho+\mathrm{div}\,(\rho\mathbf{v})=0; \tag{4}$$

(2) the balance of linear momentum (I)

$$\rho\frac{d\mathbf{v}}{dt}+\mathrm{div}\,\mathbf{\Pi}=\mathbf{0}, \tag{5}$$

with $\mathbf{\Pi}$, the dyadic flux of linear momentum, given as

$$\mathbf{\Pi}=p\boldsymbol{\delta}+\mathbf{\Pi}^{\mathrm{visc}}=p\boldsymbol{\delta}-2\eta\,\mathrm{d\acute{e}f}\,\mathbf{v}-\zeta\,\mathrm{div}\,\mathbf{v}\,\boldsymbol{\delta};$$

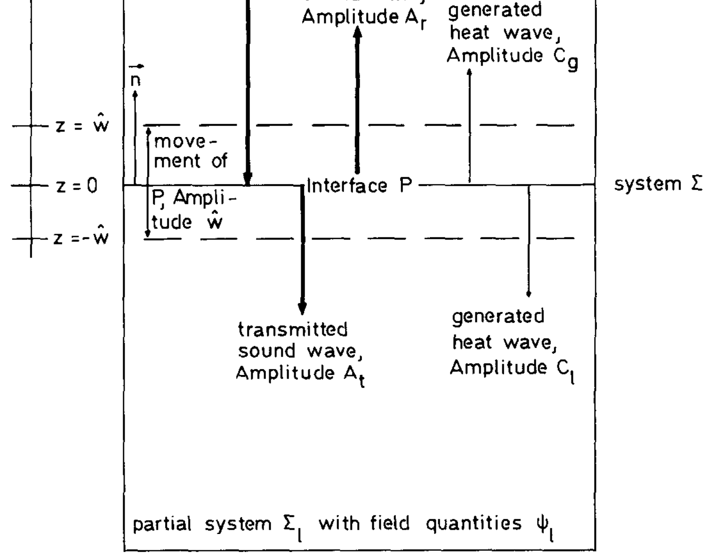

FIG. 1. Σ_g and Σ_l together with the types of waves in question.

(3) the balance of entropy (S)

$$\rho \frac{\mathrm{d}s}{\mathrm{d}t} + \operatorname{div} \frac{\mathbf{J}^{\mathrm{q}}}{T} = \mathbf{J}^{\mathrm{q}} \operatorname{grad} \frac{1}{T} - \frac{1}{T} \mathbf{\Pi}^{\mathrm{visc}} : \operatorname{grad} \mathbf{v}, \quad (6)$$

with the heat flux $\mathbf{J}^{\mathrm{q}} = -\lambda \operatorname{grad} T$ where λ denotes the thermal conductivity.

An equation which will be needed not in this chapter but in the following ones is

(4) the balance of energy (E)

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{v^2}{2} + u\right) + \operatorname{div}(\mathbf{\Pi} \cdot \mathbf{v} + \mathbf{J}^{\mathrm{q}}) = 0, \quad (7)$$

with the kinetic energy $v^2/2$ and the internal energy u. Equation (7) is noted for the sake of completeness in Table 1; it shows the field quantities used, ψ, their flux densities, $\mathbf{J}^{\psi}$, and their source densities, σ^{ψ}.

Linearizing ρ, p, s, and T according to equation (3) (which means that the incident sound wave must not have a great amplitude A_{i}) yields, together with the linearized equations of state

$$\tilde{\rho} = \frac{\gamma}{c^2}\tilde{p} - \alpha_p \rho_0 \tilde{T}, \quad (8)$$

$$\tilde{s} = -\frac{\alpha_p}{\rho_0}\tilde{p} + \frac{c_p}{T_0}\tilde{T}, \quad (9)$$

a homogeneous system of differential equations

$$\frac{\gamma}{\rho_0 c^2}\frac{\partial \tilde{p}}{\partial t} + \frac{\partial v}{\partial z} - \alpha_p \frac{\partial \tilde{T}}{\partial t} = 0, \quad (10)$$

$$\frac{1}{\rho_0}\frac{\partial \tilde{p}}{\partial z} + \frac{\partial v}{\partial t} - \frac{(4/3)\eta + \zeta}{\rho_0}\frac{\partial^2 v}{\partial z^2} = 0, \quad (11)$$

$$\alpha_p T_0 \frac{\partial \tilde{p}}{\partial t} - \rho_0 c_p \frac{\partial \tilde{T}}{\partial t} + \lambda \frac{\partial^2 \tilde{T}}{\partial z^2} = 0. \quad (12)$$

Equations (10)–(12) show the advantage of calculating only perpendicular incidence of the sound wave on P. Oblique incidence would require a set of at least nine differential equations because of the three components of the velocity [transition of equation (5) to equation (11)] and the propagation of the viscous waves in both media and the surface waves along the interface. The transformation from perpendicular incidence to oblique incidence is not trivial.

Making an ansatz of the treatment for $\tilde{p}$, v, and $\tilde{T}$

$$\frac{\tilde{p}}{p_0} = A \exp(\mathrm{i}(kz - \omega t)), \quad (13)$$

$$\frac{v}{c} = B \exp(\mathrm{i}(kz - \omega t)), \quad (14)$$

$$\frac{\tilde{T}}{T_0} = C \exp(\mathrm{i}(kz - \omega t)), \quad (15)$$

one gets a biquadratic equation to determine the wave number k with four solutions $k_1^{\pm}$ and $k_2^{\pm}$

$$k_1^{\pm} \stackrel{\mathrm{def}}{=} k^{\pm} = \pm \frac{\omega}{c}\left[1 + \frac{\mathrm{i}\,\varepsilon^2}{2}\left(1 + \frac{\gamma - 1}{Pr}\right)\right], \quad (16)$$

$$\frac{k_2^{\pm}}{1+\mathrm{i}} \stackrel{\mathrm{def}}{=} \kappa^{\pm} = \pm\sqrt{\left(\frac{\omega}{2a}\right)}\left[1 + \frac{\mathrm{i}\,\varepsilon^2}{2}(\gamma - 1)\left(1 - \frac{1}{Pr}\right)\right]. \quad (17)$$

Here ε^2 denotes a parameter of the expansion

$$\varepsilon^2 \stackrel{\mathrm{def}}{=} \frac{(4/3)\eta + \zeta}{\rho_0}\frac{\omega}{c^2},$$

subject to the condition $\varepsilon^2 \ll 1$ (this is a restriction to the sound frequency),

$$Pr \stackrel{\mathrm{def}}{=} \frac{c_p \eta (4/3 + \zeta/\eta)}{\lambda},$$

a Prandtl number which is modified by an additional bulk viscosity, a is the thermal diffusivity

$$a = \frac{\lambda}{\rho_0 c_p},$$

and $k^{\pm}$ is the wave number of the sound wave, while $\kappa^{\pm}$ is the wave number associated with the thermal wave. The following solutions of equations (10)–(12) describe the propagation of sound and heat waves in Σ_{g} and Σ_{l}:

(1) in the vapour

$$\frac{\tilde{p}_{\mathrm{g}}}{p_0} = \left[A_{\mathrm{i}} \exp(-\mathrm{i}\,k_{\mathrm{g}} z) + A_{\mathrm{r}} \exp(\mathrm{i}\,k_{\mathrm{g}} z) - \mathrm{i}\,\gamma_{\mathrm{g}} \varepsilon_{\mathrm{g}}^2 \left(1 - \frac{1}{Pr_{\mathrm{g}}}\right) C_{\mathrm{g}} \exp(-(1-\mathrm{i})\kappa_{\mathrm{g}} z)\right] \times \exp(-\mathrm{i}\,\omega t), \quad (18)$$

$$\frac{v_{\mathrm{g}}}{c_{\mathrm{g}}} = \left[-\frac{1}{\gamma_{\mathrm{g}}}\left(1 - \frac{\mathrm{i}\,\varepsilon_{\mathrm{g}}^2}{2}\left(1 - \frac{\gamma_{\mathrm{g}} - 1}{Pr_{\mathrm{g}}}\right)\right)(A_{\mathrm{i}} \exp(-\mathrm{i}\,k_{\mathrm{g}} z) - A_{\mathrm{r}} \exp(\mathrm{i}\,k_{\mathrm{g}} z)) - \frac{1-\mathrm{i}}{\sqrt{2}}\frac{\varepsilon_{\mathrm{g}}}{\sqrt{Pr_{\mathrm{g}}}} C_{\mathrm{g}} \exp(-(1-\mathrm{i})\kappa_{\mathrm{g}} z)\right] \times \exp(-\mathrm{i}\,\omega t), \quad (19)$$

Table 1. ψ, $\mathbf{J}^{\psi}$ and σ^{ψ}

Balance equation for	ψ	$\mathbf{J}^{\psi}$	σ^{ψ}
Mass (M)	1	$\mathbf{0}$	0
Linear momentum (I)	$\mathbf{v}$	$\mathbf{\Pi}$	$\mathbf{0}$
Energy (E)	$\frac{v^2}{2} + u$	$\mathbf{\Pi} \cdot \mathbf{v} + \mathbf{J}^{\mathrm{q}}$	0
Entropy (S)	s	$\frac{\mathbf{J}^{\mathrm{q}}}{T}$	$\mathbf{J}^{\mathrm{q}} \operatorname{grad} \frac{1}{T} - \frac{1}{T}\mathbf{\Pi}^{\mathrm{visc}} : \operatorname{grad} \mathbf{v}$

$$\frac{\tilde{T}_g}{T_0} = \left[\Gamma_g\left(1-\frac{\mathrm{i}\,\varepsilon_g^2}{Pr_g}\right)(A_i \exp(-\mathrm{i}\,k_g z) + A_r \exp(\mathrm{i}\,k_g z)) + C_g \exp(-(1-\mathrm{i})\kappa_g z)\right]\exp(-\mathrm{i}\,\omega t)\,; \quad (20)$$

(2) in the liquid

$$\frac{\tilde{p}_l}{p_0} = A_t \exp(-\mathrm{i}\,k_l z)\exp(-\mathrm{i}\,\omega t), \quad (21)$$

$$\frac{v_l}{c_l} = \left[-\frac{p_0}{\rho_{10}c_l^2}A_t \exp(-\mathrm{i}\,k_l z) + \frac{1-\mathrm{i}}{\sqrt{2}}\frac{\alpha_{pl}T_0}{\sqrt{Pr_l}}\varepsilon_l C_l \exp((1-\mathrm{i})\kappa_l z)\right]\exp(-\mathrm{i}\,\omega t), \quad (22)$$

$$\frac{\tilde{T}_l}{T_0} = [\Gamma_l A_t \exp(-\mathrm{i}\,k_l z) + C_l \exp((1-\mathrm{i})\kappa_l z)] \times \exp(-\mathrm{i}\,\omega t)\,; \quad (23)$$

(3) the movement of P is described by

$$\frac{w}{c_g} = \hat{w}\exp(-\mathrm{i}\,\omega t). \quad (24)$$

To reduce the dimensions of the foregoing equations, a factor Γ was introduced by the definition

$$\Gamma \overset{\text{def}}{=} \frac{p_0}{\rho_0 c^2}\frac{\gamma-1}{\alpha_p T_0},$$

which is provided with the appropriate index g or l.

Taking a system consisting of pure water and its vapour between 0 and 100°C, ε_l^2 takes values between 10^{-8} and 10^{-9} (calculated for a sound frequency of 1000 Hz); ε_l^2 is thus in the order of magnitude of the acoustic approximation. To be consistent in the following calculations ε_l^2 has to be omitted, while ε_g^2 has to be considered ($\varepsilon_g^2 \sim 10^{-5}$).

3. BOUNDARY CONDITIONS IN TWO-PHASE SYSTEMS

In 1973 one of us published a general balance equation for an interface between two media which are in contact by mass and energy transfer [3]. With the help of this equation, boundary conditions for the system under discussion are derived. The above-mentioned equation gives a relation between the field quantities ψ and the flux densities $\mathbf{J}^\psi$ which are defined in the bulk of the two partial systems Σ_g and Σ_l and the source density τ_ψ of ψ on the interface

$$(\mathbf{J}_l^\psi - \mathbf{J}_g^\psi)^n + \rho_l\psi_l(\mathbf{v}_l^n - w) - \rho_g\psi_g(\mathbf{v}_g^n - w) - \dot{\chi}_\psi + 2Hw\chi_\psi + \tau_\psi = 0. \quad (25)$$

The additional terms χ_ψ, $\dot{\chi}_\psi$, and H denote the surface density of ψ, its temporal variation and the mean curvature of P, respectively. The superscript n denotes taking the component normal to P of $\mathbf{J}^\psi$ and $\mathbf{v}$.

Making some assumptions about the system under discussion, equation (25) is simplified slightly:

(a) P is infinitely extended and plane which means that

$$H = 0.$$

(b) Σ has no surface tension which leads to

$$\chi_E = 0, \qquad \chi_S = 0,$$

$$\dot{\chi}_E = 0, \qquad \dot{\chi}_S = 0.$$

(c) There is no accumulation of mass on P. This leads to

$$\chi_M = 0, \qquad \chi_I = 0,$$

$$\dot{\chi}_M = 0, \qquad \dot{\chi}_I = 0.$$

(d) There is no production of any field quantity on P except that of the entropy s; furthermore, there are no external forces acting on Σ which leads to

$$\tau_M = 0; \quad \tau_I = 0; \quad \tau_E = 0;$$

but

$$\tau_S \neq 0.$$

These claims lead to five surface conditions which are derived from equations (4)–(7). Three surface conditions are contributed by the balance of mass, linear momentum, and energy, while the balance of entropy yields two surface conditions because of the linear independence of mass and energy fluxes.

The balance of mass yields [by setting in the appropriate ψ and $\mathbf{J}^\psi$ from Table 1 in equation (25)]

$$\rho_g(v_g - w) = \rho_l(v_l - w). \quad (26)$$

Linearizing equation (26) leads to

$$\rho_{g0}(v_g - w) = \rho_{l0}(v_l - w). \quad (27)$$

Defining the mass flux across P

$$J_M = \dot{m} \overset{\text{def}}{=} \rho_g(v_g - w), \quad (28)$$

and substituting equation (28) in equation (25) one gets [together with conditions (a)–(d)] a very simple form of equation (25)

$$(\mathbf{J}_l^\psi - \mathbf{J}_g^\psi)^n + \dot{m}(\psi_l - \psi_g) + \tau_\psi = 0. \quad (29)$$

For vanishing mass flux $\dot{m}$, equation (26) is the first of the two classical boundary conditions, $v_g = v_l$.

It is to be stressed that only in the case of entropy balance one obtains $\tau_\psi \neq 0$. The balance of linear momentum yields (already in linearized form)

$$\tilde{p}_g - ((4/3)\eta_g + \zeta_g)\frac{\partial}{\partial z}(v_g - w) = \tilde{p}_l. \quad (30)$$

The asymmetric form of equation (30) is explained by the fact that the derivative $(\partial/\partial z)(v_l - w)$ on the RHS of equation (30) yields terms of the order of ε_l^2; to remain consistent in the approximation these terms have to be omitted. Equation (30) is the second of the classical boundary conditions, if the viscosities are neglected.

The balance of energy yields

$$J_E = -\lambda_l \frac{\partial T_l}{\partial z} + \dot{m}\Big[h_l + (1/2)(v_l - w)^2 - \frac{1}{\rho_l}((4/3)\eta_l + \zeta_l)\frac{\partial}{\partial z}(v_l - w)\Big], \quad (31a)$$

with the specific enthalpy h. In equation (31a), the LHS has been defined as the flux of energy across P

$$J_E \stackrel{\text{def}}{=} -\lambda_g \frac{\partial T_g}{\partial z} + \dot{m}\Big[h_g + (1/2)(v_g - w)^2 - \frac{1}{\rho_g}((4/3)\eta_g + \zeta_g)\frac{\partial}{\partial z}(v_g - w)\Big]. \quad (31b)$$

Linearizing equations (31a) and (31b) yields

$$\lambda_g \frac{\partial \tilde{T}_g}{\partial z} - \lambda_l \frac{\partial \tilde{T}_l}{\partial z} = \dot{m}_1 L_0, \quad (32)$$

with the heat of condensation L_0, the enthalpy $h[h = (p/\rho) + u]$, and the linearized mass flux

$$\dot{m}_1 \stackrel{\text{def}}{=} \rho_{g0}(v_g - w)$$

(the subscript 1 denotes that $\dot{m}_1$ is a first-order quantity).

The next boundary condition is due to the balance of entropy

$$\tau_S = \dot{m}(s_g - s_l) + \frac{J_g^q}{T_g} - \frac{J_l^q}{T_l}. \quad (33)$$

From this equation, the conjugated forces of the fluxes J_M and J_E, X_M and X_E can be obtained. Thus

$$X_M = \Delta\Big\{-\frac{1}{T}\Big[\mu + \frac{1}{2}(v - w)^2 - \frac{1}{\rho}((4/3)\eta + \zeta)\frac{\partial}{\partial z}(v - w)\Big]\Big\}, \quad (34a)$$

with the chemical potential μ and

$$X_E = \Delta\Big\{\frac{1}{T}\Big\}, \quad (34b)$$

because of the validity of the relation

$$\tau_S = J_M X_M + J_E X_E. \quad (35)$$

The notation $\Delta\{\psi\}$ indicates that the difference of the field quantities ψ, $\psi_g - \psi_l$ has to be taken. Fluxes and their conjugated forces are (according to Onsager) combined as follows

$$J_M = L_{MM} X_M + L_{ME} X_E, \quad (36a)$$

$$J_E = L_{EM} X_M + L_{EE} X_E. \quad (36b)$$

The relation $L_{ME} = L_{EM}$ must hold.

In linearized form the two boundary conditions read [after comparing equations (28) and (31b) with equations (36a) and (36b) with the X_M and X_E inserted]

$$\rho_{g0}(v_g - w) = \frac{L_{MM}(1 - a_\rho)}{T_0 \rho_{g0}}\Big[\frac{d\pi}{dT}\tilde{T}_l - \tilde{p}_g + ((4/3)\eta_g + \zeta_g)\frac{\partial}{\partial z}(v_g - w)\Big] - \frac{L_{12}}{T_0}\Big[\frac{\tilde{T}_g}{T_0} - \frac{\tilde{T}_l}{T_0}\Big], \quad (37a)$$

$$-\lambda_g \frac{\partial \tilde{T}_g}{\partial z} = \frac{L_{12}(1 - a_\rho)}{T_0 \rho_{g0}}\Big[\frac{d\pi}{dT}\tilde{T}_l - \tilde{p}_g + ((4/3)\eta_g + \zeta_g)\frac{\partial}{\partial z}(v_g - w)\Big] - \frac{L_{22}}{T_0}\Big[\frac{\tilde{T}_g}{T_0} - \frac{\tilde{T}_l}{T_0}\Big]. \quad (37b)$$

Here $d\pi/dT$ denotes the Clausius–Clapeyron equation

$$\frac{d\pi}{dT} = \frac{L_0}{T_0(1/\rho_{g0} - 1/\rho_{l0})},$$

where a_ρ the ratio of the densities of the liquid and its vapour ($a_\rho \ll 1$, but not negligible in the case of our calculations), and L_{12} and L_{22} are abbreviations for the somewhat lengthy combinations of Onsager coefficients

$$L_{12} \stackrel{\text{def}}{=} L_{EM} - h_{g0} L_{MM},$$

$$L_{22} \stackrel{\text{def}}{=} L_{EE} - h_{g0}(L_{EM} + L_{ME}) + h_{g0}^2 L_{MM}.$$

Neglecting the movement of P and the viscosities in equations (37a) and (37b), the Onsager coefficients used here and the coefficients L_{PP}, L_{TT}, L_{PT}, and L_{TP} used in ref. [2] are connected as follows

$$L_{PP} = \frac{T_0(L_{EE} - h_{g0}(L_{EM} + L_{ME}) + h_{g0}^2 L_{MM})}{(1 - a_\rho)(L_{MM}L_{EE} - L_{EM}L_{ME})},$$

$$L_{TT} = \frac{c_g p_0 T_0 L_{MM}}{\gamma_g(L_{MM}L_{EE} - L_{EM}L_{ME})},$$

$$L_{PT} = \frac{-\rho_{g0} c_g T_0 (L_{ME} - h_{g0} L_{MM})}{\gamma_g(1 - a_\rho)(L_{MM}L_{EE} - L_{EM}L_{ME})};$$

$$L_{TP} = (1 - a_\rho)L_{PT}.$$

The factors L_{PT} and L_{TP} are not symmetric, they differ by a factor $(1 - a_\rho)$. However, the reciprocity relation of Onsager must hold.

4. ACOUSTIC COEFFICIENTS

Inserting equations (18)–(24) in the boundary conditions, equations (27), (30), (32), (37a) and (37b) one gets (for $z = 0$) an inhomogeneous system of five equations to determine the five unknown acoustic coefficients D_r, D_t, W_g, W_l, and Z. These acoustic coefficients are the amplitudes of the reflected, transmitted, and transformed waves, related to the amplitude of the incident sound wave

$$D_r = \frac{A_r}{A_i}, \quad D_t = \frac{A_t}{A_i}, \quad W_g = \frac{C_g}{A_i},$$

$$W_l = \frac{C_l}{A_i}, \quad Z = \frac{\hat{w}}{A_i}.$$

The coefficients of reflection and transmission of the sound wave are

$$D_r = \frac{1-a_\rho a_c}{1+a_\rho a_c}\left(1+\frac{1-i}{\sqrt{2}}\frac{\varepsilon_g}{\sqrt{Pr_g}}D_r^{(1)}\right), \quad (38a)$$

$$D_t = \frac{2}{1+a_\rho a_c}\left(1+\frac{1-i}{\sqrt{2}}\frac{\varepsilon_g}{\sqrt{Pr_g}}D_t^{(1)}\right). \quad (38b)$$

For the sake of simplicity of the calculations (all calculations had to be done by hand) only correction terms of the order of magnitude of ε_g are considered. The correction terms are [the superscript (1) means the first-order correction term of D_r, etc.]

$$D_r^{(1)} = 2\gamma_g(1-a_\rho)\left(\frac{p_0}{T_0}-\Gamma_g\frac{d\pi}{dT}\right)\left(\frac{c_{pg}T_0}{L_0}-\frac{1}{1-a_\rho}\right) \times \frac{1}{(d\pi/dT)(1-a_\rho^2a_c^2)}, \quad (38c)$$

$$D_t^{(1)} = \gamma_g(1-a_\rho)\left(\frac{p_0}{T_0}-\Gamma_g\frac{d\pi}{dT}\right)\left(\frac{c_{pg}T_0}{L_0}-\frac{1}{1-a_\rho}\right) \times \frac{1}{(d\pi/dT)(1+a_\rho a_c)}. \quad (38d)$$

Neglecting the heat conduction in the vapour, one gets the classical acoustic coefficients D_r and D_t [1, 4].

The transformation coefficients W_g and W_l are formed like D_r and D_t which means that they consist of one zero-order and one first-order term in ε_g (and several other terms of higher order in ε_g, which are omitted here for the above-mentioned reasons) but for the sake of brevity only the zero-order term will be noted

$$W_g = \frac{2((p_0/T_0)-\Gamma_g(d\pi/dT))}{(d\pi/dT)(1+a_\rho a_c)}, \quad (39a)$$

$$W_l = \frac{2((p_0/T_0)-\Gamma_l(d\pi/dT))}{(d\pi/dT)(1+a_\rho a_c)}, \quad (39b)$$

$$Z = -\frac{2a_\rho a_c}{\gamma_g}. \quad (39c)$$

At first sight it seems to be astonishing that the zero-order term of W_g does not vanish if λ_g is set to zero. But a closer examination of the governing equations reveals that the heat wave in the vapour is suppressed for $\lambda_g \to 0$. The transition $\lambda_g \to 0$ produces $\kappa_g \to \infty$ which means that in equations (18) and (19) the heat wave in the gas is damped out. Furthermore, only four boundary conditions can be deduced, for the balance of entropy yields only one boundary condition because of the linear dependence of J_M and J_E.

The boundary condition for $\lambda_g \to 0$ is

$$\rho_{g0}(v_g-w) = \frac{L_{MM}(1-a_\rho)}{T_0\rho_{g0}} \times\left[\frac{d\pi}{dT}\tilde{T}_l-\tilde{p}_g+((4/3)\eta_g+\zeta_g)\frac{\partial}{\partial z}(v_g-w)\right], \quad (40)$$

which means that only one Onsager coefficient remains to be determined and that there is no possibility of calculating W_g. The omission of the heat wave in the gas while preserving the old boundary conditions as done in ref. [2] is doubtful.

5. DISSIPATION OF SOUND ENERGY

To determine the time-averaged sound energy which is dissipated on P, the fluxes of sound energy arriving at and departing from P are added up and averaged. Taking into account the movement of P this method yields instantaneously [4]

$$\overline{X} = \overline{\tilde{p}_l(v_l-w)}-\overline{\tilde{p}_g(v_g-w)}. \quad (41)$$

Here, $\overline{X}$ denotes the amount of sound energy which is dissipated on average.

Before inserting equations (18), (19), (21), (22), and (24) into equation (41) it is to be considered that nonlinear operations between harmonic waves require a return from complex representation of the waves to the real one. Thus, the real parts are to be taken from the amplitudes A_r, A_t, C_g, and $\hat{w}$ and from the time-dependent parts of the exponential function [5]; the position-dependent parts vanish for the calculations are done for $z = 0$ in a rough approximation although the movement of P is considered

$$\begin{aligned}\overline{X} = {} & \overline{\mathrm{Re}\{p_0A_t\exp(-i\,\omega t)\}} \\ & \times\overline{\mathrm{Re}\left\{-\frac{p_0}{\rho_{10}c_l}A_t\exp(-i\,\omega t)-\hat{w}c_g\exp(-i\,\omega t)\right\}} \\ & -\overline{\mathrm{Re}\{p_0A_i\exp(-i\,\omega t)+p_0A_r\exp(-i\,\omega t)\}} \\ & \times\overline{\mathrm{Re}\left\{-\frac{c_g}{\gamma_g}A_i\exp(-i\,\omega t)+\frac{c_g}{\gamma_g}A_r\exp(-i\,\omega t)\right.} \\ & \overline{\left.-c_g\frac{1-i}{\sqrt{2}}\frac{\varepsilon_g}{\sqrt{Pr_g}}C_g\exp(-i\,\omega t)-\hat{w}c_g\exp(-i\,\omega t)\right\}},\end{aligned} \quad (42)$$

with

$$\exp(-i\,\omega t) = \cos(\omega t)-i\sin(\omega t), \quad (43a)$$

$$A_r = A_{rr}+i\,A_{ri}, \quad (43b)$$

$$A_t = A_{tr}+i\,A_{ti}, \quad (43c)$$

$$C_g = C_{gr}+i\,C_{gi}, \quad (43d)$$

$$\hat{w} = \hat{w}_r+i\,\hat{w}_i. \quad (43e)$$

Here, the second subscript denotes, respectively, the real and imaginary part of the amplitudes (A_i is considered to be totally real). Taking the time-averaged value one gets a factor of 1/2 only for those terms which are of the order $\sin^2(\omega t)$ or $\cos^2(\omega t)$; mixed terms vanish. Dividing $\overline{X}$ by $(1/2)(p_0^2A_i^2/\rho_{g0}c_g)$, the connection with the above calculated acoustic coefficients is made

$$\frac{\bar{X}}{(1/2)(p_0^2A_i^2/\rho_{g0}c_g)} = \underline{1} - (\underline{D_{rr}^2} + \underline{D_{ri}^2})$$
$$- a_\rho a_c(D_{tr}^2 + D_{ti}^2) + \gamma_g Z_i(D_{ri} + D_{ti})$$
$$+ \frac{\gamma_g}{\sqrt{2}} \frac{\varepsilon_g}{\sqrt{Pr_g}} [(W_{gr} + W_{gi})(1 + D_{rr})$$
$$+ (W_{gi} - W_{gr})D_{ri}] + \gamma_g Z_r(\underline{1 + D_{rr} - D_{tr}}). \quad (44)$$

The second subscripts r and i denote the real and imaginary parts of the acoustic coefficients, respectively. In the case of classical acoustics only the underlined (dashed line) coefficients exist (if the movement of P is considered); taking their values from equations (38) and (39c) and inserting them into equation (44) one gets

$$\left[\frac{\bar{X}}{(1/2)(p_0^2A_i^2/\rho_{g0}c_g)}\right]_{\text{class.}} = 1 - \left(\frac{1 - a_\rho a_c}{1 + a_\rho a_c}\right)^2$$
$$- a_\rho a_c\left(\frac{2}{1 + a_\rho a_c}\right)^2 - 2a_\rho a_c\left(1 - \frac{1 - a_\rho a_c}{1 + a_\rho a_c}\right.$$
$$\left. + \frac{2}{1 + a_\rho a_c}\right) = 0. \quad (45)$$

This result was to be expected; the energy dissipation in the classical case is zero. For our case the calculation of $\bar{X}$ yields (if only terms of the order of ε_g are taken into account)

$$\frac{\bar{X}}{(1/2)(p_0^2A_i^2/\rho_{g0}c_g)} = \varepsilon_g\sqrt{\left(\frac{2}{Pr_g}\right)}$$
$$\times \left[D_r^{(1)}\left(2\left(\frac{1 - a_\rho a_c}{1 + a_\rho a_c}\right)^2 + 2a_\rho a_c\frac{1 - a_\rho a_c}{1 + a_\rho a_c}\right)\right.$$
$$+ D_t^{(1)}\, 2a_\rho a_c\left(\left(\frac{2}{1 + a_\rho a_c}\right)^2 - \frac{2}{1 + a_\rho a_c}\right)$$
$$\left. + \gamma_g\frac{2}{1 + a_\rho a_c}\,\frac{(p_0/T_0) - \Gamma_g(\mathrm{d}\pi/\mathrm{d}T)}{(\mathrm{d}\pi/\mathrm{d}T)(1 + a_\rho a_c)}\right]. \quad (46)$$

For a system consisting of water and its vapour and a sound frequency of 1000 Hz, $\bar{X}$ is a purely temperature-dependent quantity; Table 2 shows some numerical values of $\bar{X}$ which are calculated with the help of thermodynamic tables. Unfortunately, Table 2 could not be verified because of a lack of measurements published in the literature.

Table 2. Percentage dissipation of energy dependent on T

T_0 (°C)	Mean dissipation of energy (%)
0	0.192
10	0.118
20	0.0841
30	0.0610
40	0.0449
50	0.0337
60	0.0256
70	0.0197
80	0.0153
90	0.0120
100	0.00914

6. SUMMARY

The balance equations for mass, linear momentum, energy, and entropy yield a set of differential equations which have solutions which describe the propagation of sound and temperature waves in a two-phase system. The above-mentioned equations also provide a set of five boundary conditions with the help of a general balance equation for interfaces. They permit a calculation of acoustic coefficients taking into account mass and energy transport across the interface. Finally, the dissipation of energy on P is calculated.

7. ADDITIONAL REMARK

As we recently got to know, equations (16) and (17) were found already by Epstein and Carhart in 1953 [6].

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RÉFLEXION D'UNE ONDE SONORE À UNE SURFACE ENTRE DEUX PHASES EN CONSIDÉRATION DU TRANSPORT DE LA MATIÈRE ET DE LA CHALEUR

Résumé—Des conditions aux limites pour un système hydrodynamique à deux phases sont établies à l'aide d'une équation dérivée des principes généraux. Avec ces conditions on peut calculer les coefficients de réflexion et de transmission d'une onde sonore et, de plus, les coefficients de transformation qui donnent les amplitudes des ondes de chaleur se propageantes dans la vapeur et dans le liquide. La dissipation de l'énergie est calculée.

REFLEXION VON SCHALLWELLEN AN EINER PHASENGRENZFLÄCHE UNTER BERÜCKSICHTIGUNG VON MASSEN- UND WÄRMETRANSPORT

Zusammenfassung—Mit Hilfe einer aus allgemeinen physikalischen Prinzipien abgeleiteten Gleichung (Meinhold-Heerlein, 1973) werden Randbedingungen für ein Zweiphasensystem, das Schallwellen reflektiert, erstellt, die unter anderem die beiden Randbedingungen der klassischen Akustik, nämlich Stetigkeit des Druckes und der Normalkomponente der Schallschnelle über die Phasengrenzfläche als Grenzfall enthalten. Aus den o.a. Randbedingungen werden Reflexions- und Transmissionskoeffizienten berechnet, die als führende Terme die klassischen akustischen Koeffizienten enthalten. Darüber hinaus werden 3 Transformationskoeffizienten angegeben, die die Ausbreitung von Wärmewellen im Dampf- und Flüssigkeitshalbraum sowie die Bewegung der Phasengrenzfläche beschreiben. Die bei der Reflexion der Schallwelle auftretende Energiedissipation wird (im zeitlichen Mittel) berechnet.

ИССЛЕДОВАНИЕ ОТРАЖЕНИЯ ЗВУКА ОТ ГРАНИЦЫ РАЗДЕЛА С УЧЕТОМ ПРОИСХОДЯЩИХ НА НЕЙ ПРОЦЕССОВ МАССО- И ТЕПЛОПЕРЕНОСА

Аннотация—С помощью уравнения, предложенного Майнхольдом и Хеерлайном в 1973 году, выведено пять граничных условий для двухфазной системы. Они включают два граничных условия классической акустики, а именно: постоянство давления и нормальной компоненты скорости на границе раздела, что позволяет рассчитать коэффициенты отражения и пропускания, в которые в качестве основных членов входят классические акустические коэффициенты. Кроме того, приведены три коэффициента преобразования, описывающих движение границы раздела и распространение тепловых волн в жидкости и паре. Рассчитана усредненная во времени диссипация энергии.